Scientific African 8 (2020) e00427

Contents lists available at ScienceDirect

Scientific African

journal homepage: www.elsevier.com/locate/sciaf

Mathematical analysis of affinity hemodialysis on T-Cell depletion

A.M. Okedoye[a], S.O. Salawu[b,*], S.I. Oke[c], N.K. Oladejo[b]

[a] Department of Mathematics and Computer Science, Federal University of Petroleum Resources, Effurun, Nigeria
[b] Department of Mathematics, Landmark University, Omu-aran, Nigeria
[c] Department of Mathematical Sciences, University of Zululand South Africa

ARTICLE INFO

Article history:
Received 20 January 2020
Revised 8 April 2020
Accepted 10 May 2020

Keywords:
Hemodialysis
Infection
Vaccines
Human immunodeficiency virus

ABSTRACT

Investigation into the mathematical analysis of affinity hemodialysis on T-cell depletion is considered to give more significant understanding on the infection dynamics of HIV. Our model revealed the possibility of more than two infected equilibriums with the incorporation of recovery through the affinity hemodialysis. The conditions for stable infected equilibrium to prevent full-blown of AIDS are obtained and stated as hypothesis. This work has, therefore, open the way for partnerships among modelers and clinicians to strengthening the insight into the nature and process of viral multiphasic decay noticed in some treated patients. The initial growth of virus, infected cells and glycoprotein as well as possibility of total viral clearance are examined in the study.

Introduction

The Human Immunodeficiency Virus (HIV) is a retrovirus that causes infections, which resist natural immunity is called HIV. This over some time resulted into Acquired Immuno- deficiency Syndrome (AIDS), a state in humans when there is a continuous reduction in the body immune system, Lisowska et al. [1,2]. As a result, the body system is open to life threatening infections of any kind. For decades, the scientists have been developing different Antiviral for the eliminating or treating of viral infections. Vaccines were the early adopted approach for killing or weakling of viruses which was assumed to be effective. Vaccines for lingering infections are rare to get, for instance HBV vaccine. Presently, positive vaccines have been developed for protracted infections triggered by greatly mutable viruses. HIV produces various quasi-species and several synthesized virus mutations. The quest for an active vaccine is in its second decade [3-5].

Although for long term expectation, vaccines are taken to be the best in healing chronic viral diseases. The results till this period have been unsatisfactory on vaccine treatment of viral diseases, even though the hope of long term treatment of chronic viral diseases may be vaccine. The uses of small molecule drug for the conventional therapies is steadily gaining substantial attention, mostly those focused on AIDS and HIV infection [6-10]. Despite the advances in the uses of HIV active antiretroviral therapy (HAART) in the treatment of HIV, latest investigations have revealed that some viruses continue to

* Corresponding author.
E-mail address: kunlesalawu2@gmail.com (S.O. Salawu).

https://doi.org/10.1016/j.sciaf.2020.e00427

live years after treatment in the system according to Ramratnam et al. [11]. Recently, studies have shown that affected macrophages and cells offers resistant treatment chamber that is long existed, Aquaro et al. [12].

The viral antigens generated as a result of antibodies from the body binds to the physical arbitrate clearance, will in turn decreases the infection reoccurrence rate. Also, immune system cleared the earlier affected cells to become quiescent or dies. When enough quantities of pathogen are removed from the blood, there is receding of the viral infection. Models recently show the early infection and the institution of the steady state of virus are described in [13-15]. The early stage HIV infection explicit model was provided by Stafford et al. [16]. In the model, the asymptomatic phase or the set point was established in which virus-related particle generation is in steady state with virus-related particle clearance.

The glycoprotein that envelope GP120 is the HIV, enveloped the exposed glycoprotein surface that plays an essential role in HIV treatment. The GP120 is gotten from its molecular weight, it is significant for infection entrance into the cells as it is important in connection to the particular cell receptor surface. This is necessary to CD4 induces is the conformation start of a flow of vicissitudes in GP41 and GP120 that prompted to the fusion of the membrane of the host cell and viral. Tullis et al. [17] suggested a synthetic lymph node to enhance the immune system in combating viral infections. They show theoretically that the method could efficiently eliminate viral infectious nucleic acids, viral toxic proteins and virus from the blood, thus reduces viruses by averting reinfection of fresh cells. The work of Hildeman et al. [18] reported on the significance of reactive oxygen species in determining the fate of the stimulated T-cell. Given an explanation of the reason why most stretched T-cells die and some live to give an insight on how lymphoid cancer and autoimmunity are disallowed, and how acquired immunity in T-cells responded to stimulation of antigen through the course of differentiation, activation and division to produce a large T-cells activated effector.

Véronique et al. [19] considered the effect and the mechanism of Apoptosis, or death programmed cell on the steady T-cell failure that arises in the patients suffering from HIV infection. The results obtained encapsulates the present understanding of Env-mediated death cell resulting from T-cell weakening and clinical difficulties as well as contradictory studies that address the likely mechanisms of the death of T-cells. While Ayeni et al. [20] examined the impact of hemodialysis affinity on AIDS/HIV as a possible alternative for the treatment of HIV patients that is drug resistant. It was also shown that hemodialysis affinity is supposedly suitable adjunctive therapies that can be used in HIV infected patient's treatment either directly or in combination with drug treatment.

To estimate the impacts of HIV infection on precursors intrathymic of T-cells in adults infected by HIV, Bandera et al. [21] carried out a comprehensive study of the immune-phenotypic thymic tissue separated from negative 10 HIV adults and 7 HIV infected individuals who were to undertake heart operation. It was noticed that thymuses of HIV carriers were made up of relatively depleted CD4+ solitary positive T-cells and an equivalent enhancement of CD8+ single positive T-cells. The obtained results revealed the rate at which thymopoiesis is generated, the HIV infected thymic tissues subjects are composed of thymocyte hyperactivation immune concerning either active T-cells undergoing thymopoiesis or mature T-cells. Doitsh et al. [22] shows that apoptosis caspase-3 only responsible for the death of a little portion of the effectively infected cells while the remaining 95 percent of inactive lymphoid CD4+ *T*-cells die as a result of pyroptosis caspase-1 prompted by failed virus-related infection. Pyroptosis relates to an extremely inflammatory form of programmed cell death in which cytokines pro-inflammatory and cytoplasmic contents along with IL-1β are unrestricted. In the work of Keck et al. [23], the separate influences of some terms on the differentiation of CD4+ *T*-cell during contamination were presented. It was observed that huge antigen affinity is absolutely associated with differentiation of T-helper (Th) at both low and high measures of antigen.

Hojo-Souza et al. [24] in their paper, reported that apoptosis CD4+ *T*-cells in Plasmodium vivax contamination are arbitrate by activation of extrinsic and intrinsic pathways. The existence of apoptosis CD4+ *T*-cells and its pathways was examined on individuals in an endemic area from (Porto Velho – RO) with obviously Plasmodium vivax infection. Their results suggested that Plasmodium vivax infection prompted CD4+ *T*-cells apoptosis by two different kinds of signaling: by decreasing the manifestation of the protein anti-apoptotic BCl-2 (intrinsic pathway), and by encouraging the death TNFR1 receptor (extrinsic pathway). While in the same year, Martinez and Evavold [25] considered the biophysical and kinetic terms of the peptide: MHC (pMHC) and T-cell receptor (TCR) interface defined the required intrinsic factors derived and activated T-cell. The mechanisms that conserve various range of high and low T-cells affinity is also discussed. Konagaya et al. [26] examined under different physiological conditions the dynamic configuration of E12A by both amide proton relaxation and exchange.

In this study, mathematical Analysis of Affinity Hemodialysis on T-Cell Depletion will be considered for the dynamics of HIV infection. We shall examine the both disease free and infection equilibrium. The stability of the infected point will be examined to ensure that glycoprotein is prevented from unnecessary increase and the stem cell is not allowed to deplete.

Existing mathematical models

The relevant existing models include;

(i) Tullis et al. [17]:

$$\frac{dT(t)}{dt} = \lambda - dT - kTV \tag{A1.1}$$

$$\frac{dT^*(t)}{dt} = kTV - \delta T^* \tag{A1.2}$$

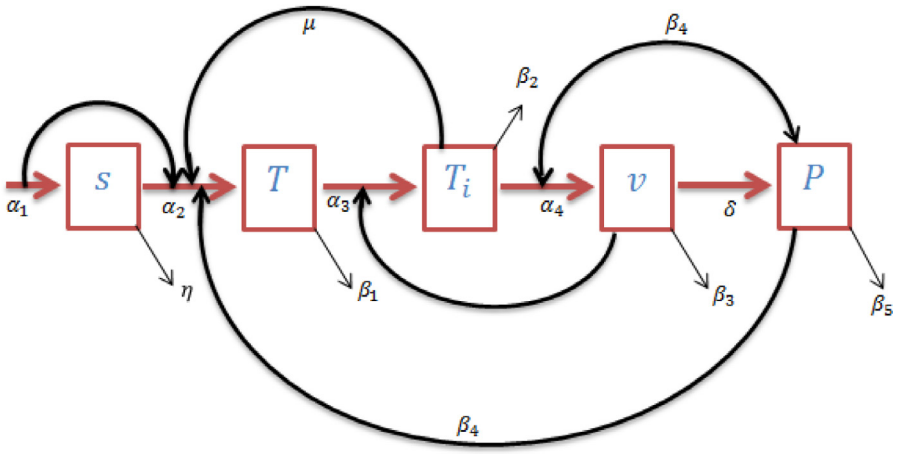

Fig. 1. The Model Dynamics.

$$\frac{dV(t)}{dt} = \pi T^* - cV \tag{A1.3}$$

(ii) Ayeni et al. [20]:

$$\frac{dT(t)}{dt} = \pi - d_1 T - k_1 TV + \mu T_i, \;\; T(0) = T_0 \tag{B2.1}$$

$$\frac{dT_i(t)}{dt} = k_1 TV - d_2 T_i - \mu T_i, \;\; T_i(0) = 0 \tag{B2.2}$$

$$\frac{dV(t)}{dt} = k_2 T_i - cV \;\; V(0) = 0 \tag{B2.3}$$

Though the mentioned models promoted dynamics of HIV infection on which the present is built on to enhance understanding of biological HIV. Nevertheless, in the models, the recovery rate and immune response was not captured in the studied.

Current modeling affinity hemodialysis on T-Cell depletion

Follow from [17] and [20] model with the incorporation of recovery through the affinity hemodialysis:

Production
$S \xrightarrow{\alpha_1}$ Availability/Production of Stem Cell
$S \xrightarrow{\alpha_2} T$ T-cell generation rate from stem cells
$T + V \xrightarrow{\alpha_3} T_i$ T-cells infection
$T_i \xrightarrow{\alpha_4} V + P$ Virus production and GP120
$P + T \overset{\alpha_5}{\leftrightarrow} PT$ GP120 reversibility on normal T-Cells
μT-cells infected recovery due to hemodialysis
Clearance
$T \xrightarrow{\beta_1}$ Death rate of normal T-cells
$T_i \xrightarrow{\beta_2}$ Rate of clearance of infected T-cell
$v \xrightarrow{\beta_3}$ Clearance rate of viral
$P \xrightarrow{\beta_5}$ glycoproteins clearance rate
$PT \xrightarrow{\beta_6}$ Glycoproteins 120 induced apostosis and clearance rate
η Stem cell death rate

Where S is the stem cell, T is Healthy T cell, P is GP120 concentration, v is virus and T_i is infected T-cells.

With reference to Fig. 1, Stem cell S is produced at rate α_1 from where uninfected T cell is produced at rate α_2, and increases by recovery of infected cells. The death of bound between healthy cell and, glycoproteins and virus depletes the availability of healthy T cells. Uncontaminated cells can become contaminated by virus at a rate α_3 to generate infected cell

cells T_i, the infected cell is depleted by virus death or death of the infected cell as well as recovery due to treatment. The viral load is produced from infected cells and bounded to uninfected cell, the virus is reduced by clearance or death at rate β_3. Glycoproteins 120 is enhanced by viral concentration and depleted by death or clearance at a rate β_5

Arising from above, the relevant mathematical equations describing the dynamics of affinity hemodialysis of T-Cells depletion are according to [17] and [20]:

$$\frac{d}{dt}S(t) = \alpha_1 S(t)\left(1 - \frac{\alpha_2}{\alpha_1}T(t)\right) - \eta S(t) \tag{1}$$

$$\frac{d}{dt}T(t) = \alpha_2 S(t)T(t) - \beta_4 P(t)T(t) - \alpha_3 \nu(t)T(t) - \beta_1 T_i(t) + \mu T_i(t) - \alpha_3 T_i(t) \tag{2}$$

$$\frac{d}{dt}T_i(t) = \alpha_3 \nu(t)T(t) - \mu T_i(t) - \beta_2 T_i(t) - \beta_3 \nu(t) \tag{3}$$

$$\frac{d}{dt}\nu(t) = \alpha_4 T_i(t) - \beta_3 \nu(t) \tag{4}$$

$$\frac{d}{dt}P(t) = \delta\nu(t) - \beta_5 P(t) \tag{5}$$

$$S(0) = 500,\ T(0) = 300,\ T_i(0) = 0,\ \nu(0) = 0,\ P(0) = 50 \tag{6}$$

Equations (1) – (6) is a modification to the existing mathematical models. The current model modifies models by (i) and (ii) by the inclusion of assumption that with treatment through dialysis, infected cells recovers to become normal cells and become healthy which in turn lowers GP 120 levels provided $\alpha_1 \neq 0$ and $\alpha_1 > \eta$ which is the conditions that guarantee the existence of stem cells.

Stability analysis

We give the following definition on stability theory of the dynamical system (1) – (6).

Definition: (a, b) is a critical point of the autonomous system

$$x' = F(x,\ y)y' = G(x,\ y)$$

when $F(a, b) = 0$ and $G(a, b) = 0$.

We write

$$\frac{dS}{dt} = f_1(\emptyset(t),\ t),\ \frac{dT}{dt} = f_2(\emptyset(t),\ t),\ \frac{dT_i}{dt} = f_3(\emptyset(t),\ t),\ \frac{d\nu}{dt} = f_4(\emptyset(t),\ t),\ \frac{dP}{dt} = f_5(\emptyset(t),\ t)$$

Where

$$\emptyset(t) = (S(t),\ T(t),\ T_i(t),\ \nu(t),\ P(t))$$

$$\begin{cases} f_1 = \alpha_1 S(t)\left(1 - \frac{\alpha_2}{\alpha_1}T(t)\right) - \eta S(t) \\ f_2 = \alpha_2 S(t)T(t) - \beta_4 P(t)T(t) - \alpha_3 \nu(t)T(t) - \beta_1 T_i(t) + \mu T_i(t) - \alpha_3 T_i(t) \\ f_3 = \alpha_3 \nu(t)T(t) - \mu T_i(t) - \beta_2 T_i(t) - \beta_3 \nu(t) \\ f_4 = \alpha_4 T_i(t) - \beta_3 \nu(t) \\ f_5 = \delta\nu(t) - \beta_5 P(t) \end{cases}$$

Setting $f_i = 0,\ \forall\ i = 1,\ \cdots,\ 5$ and solve for (S, T, T_i, v, P). This gives critical (Crt) points

$$Crt_1 = (S = 0,\ T = 0,\ T_i = 0,\ \nu = 0,\ P = 0),$$

$$Crt_2 = \left(S = \frac{\beta_1}{\alpha_2},\ T = \frac{\eta + \alpha_1}{\alpha_2},\ T_i = 0,\ \nu = 0,\ P = 0\right),$$

$$Crt_3 = \left(S = 0,\ T = e_2,\ T_i = e_1\frac{\beta_3\beta_5}{\delta\alpha_2},\ \nu = -e_1\frac{\beta_5}{\delta},\ P = -e_1\right),$$

Where

$$e_1 = \frac{\delta\beta_1(\mu + \alpha_4 + \beta_2)}{\delta\beta_4(\mu + \alpha_4 + \beta_2) + \alpha_3\beta_5(\alpha_3 + \alpha_4 + \beta_2)},\ e_2 = \frac{\beta_3(\mu + \alpha_4 + \beta_2)}{\alpha_3\alpha_4}$$

We compute the Jacobean matrix J for the system, which gives

$$J := \begin{bmatrix} -\alpha_1\left(1-\frac{\alpha_2 T}{\alpha_1}\right)-\eta & S\alpha_2 & 0 & 0 & 0 \\ \alpha_2 T & -P\beta_4 + S\alpha_2 - v\alpha_3 - \beta_1 & \mu-\alpha_3 & -\alpha_3 T & -T\beta_4 \\ 0 & v\alpha_3 & -\mu-\beta_2 & T\alpha_3-\beta_3 & 0 \\ 0 & 0 & \alpha_4 & -\beta_3 & 0 \\ 0 & 0 & 0 & \delta & -\beta_5 \end{bmatrix}$$

Evaluating J at critical point Crt_1 gives

$$A1 := \begin{bmatrix} -\alpha_1-\eta & -0. & 0 & 0 & 0 \\ 0. & -\beta_1 & \mu-\alpha_3 & -0. & -0. \\ 0 & 0. & -\mu-\beta_2 & -\beta_3 & 0 \\ 0 & 0 & \alpha_4 & -\beta_3 & 0 \\ 0 & 0 & 0 & \delta & -\beta_5 \end{bmatrix}$$

The characteristics equation corresponding to $A1$ is

$$\begin{aligned}(-\lambda-\alpha_1-\eta)(-\lambda-\beta_1)(-\lambda^3-\lambda^2\mu-\lambda^2\beta_2-\lambda^2\beta_3-\lambda^2\beta_5-\lambda\mu\beta_3-\lambda\mu\beta_5 \\ -\lambda\alpha_4\beta_3-\lambda\beta_2\beta_3-\lambda\beta_2\beta_5-\lambda\beta_3\beta_5-\mu\beta_3\beta_5-\alpha_4\beta_3\beta_5-\beta_2\beta_3\beta_5\end{aligned} \tag{7}$$

Solving (7) above, we have

$$\left\{\lambda_1=-\beta_1,\ \lambda_2=-\eta-\alpha_1,\ \lambda_3=-\beta_5,\ \lambda_4=-\frac{1}{2}(\mu+\beta_2+\beta_3)+\frac{1}{2}\sqrt{(\mu+\beta_2-\beta_3)^2-4\alpha_3\beta_3},\right.$$

$$\left.\lambda_4=-\frac{1}{2}(\mu+\beta_2+\beta_3)-\frac{1}{2}\sqrt{(\mu+\beta_2-\beta_3)^2-4\alpha_3\beta_3}\right\}$$

Evaluating J at critical point *crt*2 gives

$$A2 := \begin{bmatrix} 0 & -\beta_1 & 0 & 0 & 0 \\ \alpha_1-\eta & 0 & \mu-\alpha_3 & -e_3 & -\frac{e_3\beta_4}{\alpha_3} \\ 0 & 0 & -\mu-\beta_2 & e_2-\beta_3 & 0 \\ 0 & 0 & \alpha_4 & -\beta_3 & 0 \\ 0 & 0 & 0 & \delta & -\beta_5 \end{bmatrix}$$

Where $e_3 = \frac{\alpha_3(\eta-\alpha_1)}{\alpha_2}$

The characteristics equation corresponding to $A2$ is

$$\begin{aligned}\frac{1}{\alpha_2}((\lambda+\beta_5)(\eta\alpha_3\alpha_4+\lambda^2\alpha_2+\lambda\mu\alpha_2+\lambda\alpha_2\beta_2+\lambda\alpha_2\beta_3+\mu\alpha_2\beta_3+\mu\alpha_2\beta_3-\alpha_1\alpha_3\alpha_4 \\ +\alpha_2\alpha_4\beta_3+\alpha_2\beta_2\beta_3)(\eta\beta_1-\lambda^2-\alpha_1\beta_1))\end{aligned} \tag{8}$$

Thus solving (8), gives

$$\left\{\lambda_1=-\beta_5,\ \lambda_2=\sqrt{(\eta-\alpha_1)\beta_1},\ \lambda_3=-\sqrt{(\eta-\alpha_1)\beta_1},\ \lambda_4=-\frac{1}{2}(\mu+\beta_2+\beta_3)+\frac{1}{2}\sqrt{(\mu+\beta_2-\beta_3)^2-4\alpha_3(\beta_3-e_3)},\right.$$

$$\left.\lambda_4=-\frac{1}{2}(\mu+\beta_2+\beta_3)-\frac{1}{2}\sqrt{(\mu+\beta_2-\beta_3)^2-4\alpha_3(\beta_3-e_3)}\right\}$$

And again, evaluate J at crt3 gives

$$A3 := \begin{bmatrix} \alpha_1\left(1-\frac{\alpha_2 e_2}{\alpha_1}\right)-\eta & 0 & 0 & 0 & 0 \\ \alpha_2 e_2 & \beta_4 e_1+\frac{e_1\beta_5\alpha_3}{\delta}-\beta_1 & \mu-\alpha_3 & -\alpha_3 e_2 & -e_2\beta_4 \\ 0 & -\frac{e_1\beta_5\alpha_3}{\delta} & -\mu-\beta_2 & \alpha_3 e_2-\beta_3 & 0 \\ 0 & 0 & \alpha_4 & -\beta_3 & 0 \\ 0 & 0 & 0 & \delta & -\beta_5 \end{bmatrix}$$

With caracteristics equation defined as

$$\frac{1}{\delta}\left(-\lambda+\alpha_1\left(1-\frac{\alpha_2 e_2}{\alpha_1}\right)\right)\left(\delta\lambda^4+a\lambda^3+b\lambda^2+c\lambda+d\right) \tag{9}$$

Where

$$a=\delta\cdot\left(-\left(\beta_4-\frac{\alpha_3\beta_5}{\delta}\right)\cdot e_1+\mu+\beta_1+\beta_2+\beta_3+\beta_5\right)$$

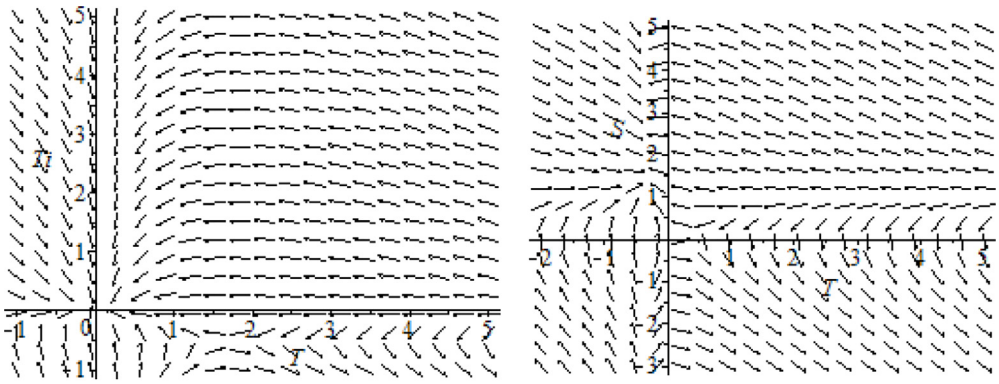

Fig. 2. Dynamics of HIV Hemodialysis Phase plot.

$$b = -\left(\alpha_3\alpha_4 e_2 + \frac{(\alpha_3+\beta_2+\beta_3+\beta_5)}{\delta}\alpha_3\beta_5 e_1\right) + (\mu+\beta_2+\beta_3+\beta_5)(\beta_1-\beta_4\cdot e_1) \\ +(\mu+\alpha_4+\beta_2+\beta_5)\beta_3 + (\mu+\beta_2)\beta_5$$

$$c := -(\delta\alpha_3\beta_1\beta_5(\mu\alpha_3\beta_3 + \mu\alpha_4\beta_3 - \mu\alpha_4\beta_5 + \mu\beta_2\beta_3 + \mu\beta_3\beta_5 + \alpha_3\alpha_4\beta_3 + \alpha_3\alpha_4\beta_5 \\ +\alpha_3\beta_2\beta_3 - \alpha_3\beta_3\beta_5 + \alpha_4^2\beta_3 + 2\alpha_4\beta_2\beta_3 + \beta_2^2\beta_3))/(\delta\mu\beta_4 + \delta\alpha_4\beta_4 + \delta\beta_2\beta_4 + \\ \alpha_3^2\beta_5 + \alpha_3\alpha_4\beta_5 + \alpha_3\beta_2\beta_5$$

$$d = -\delta\beta_1\beta_3\beta_5(\mu+\alpha_4+\beta_2)$$

Now for the eigenvalues of the associated with Crt_3, using Descartes' rule of signs, we have 1 positive root and four (4) negative roots.

Analysis of disease free and endemic equilibrium

Figs. 2(a) and (b) is a phase plot associated with disease free Fig. 2(a) and endemic Fig. 2(b) equilibrium. The phase plot shows the possible equilibrium for (T,S) and (T,T_i) respectively, for the choice of our parameters in this work. We have extended the domain to show what happens within the neighborhood of origin.

H1: Hypothesis 1

Let $\alpha_i,\ \beta_i,\ \mu,\eta\ \in \mathbb{R}^+\ \forall\ i = 1..5$, then the critical point Crt_1 associated to disease free equilibrium of the dynamical Eq. (1) – (5) is

i It is an asymptotically stable node if

$$\beta_3 + 2\sqrt{\alpha_3\beta_3} \leq \mu + \beta_2$$

ii It is an asymptotically stable spiral point if

$$\mu + \beta_2 < \beta_3 + 2\sqrt{\alpha_3\beta_3}$$

H2: Hypothesis 2

Let $\alpha_i,\ \beta_i,\ \mu,\eta\ \in \mathbb{R}^+\ \forall\ i = 1..5$, then the critical point Crt_2 associated to endemic equilibrium of the dynamical Eq. (1) – (5) is

i is stable or neutrally stable center, whenever

$$\alpha_1 > \eta$$

ii is an asymptotically stable spiral point if

$$\mu + \beta_2 < \beta_3 + 2\sqrt{\alpha_3(\beta_3 - e_3)} \text{ and } \frac{\beta_3(\mu+\alpha_4+\beta_2)}{\alpha_3\alpha_4} < 1$$

H3: Hypothesis 3

Let $\alpha_i,\ \beta_i,\ \mu,\eta\ \in \mathbb{R}^+\ \forall\ i = 1..5$, then the critical point Crt_3 associated to endemic equilibrium of the dynamical Eq. (1) – (5) is unstable for all possible of choice of $\alpha_i, \beta_i, \mu,\ \eta$.

The prove of hypothesis H1-3 follows from the solution to Eqs. (7), (8) and (9)

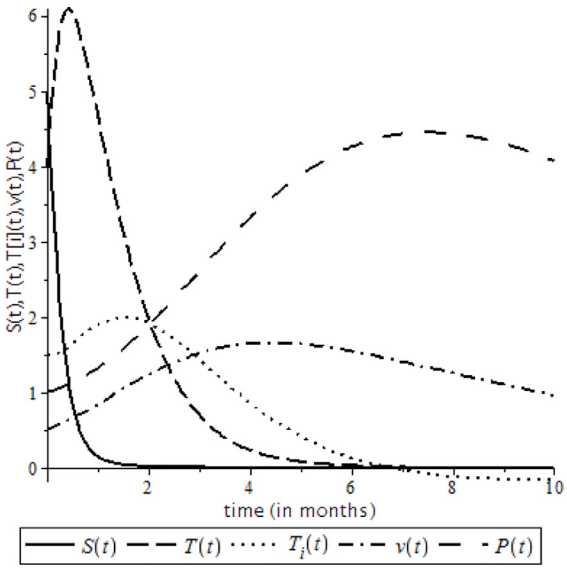

Fig. 3. Dynamics of HIV hemodialysis on T Cell depletion when$\alpha_1 = 0.1, \mu = 0.5$.

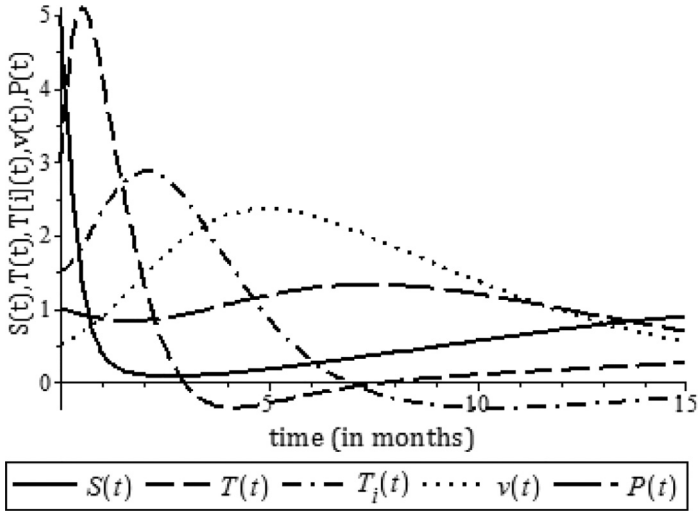

Fig. 4. Dynamics of HIV hemodialysis on T Cell depletion when $\alpha_1 = 0.5, \mu = 0.1$.

Numerical analysis

In this section, we provide some numerical analysis of the model dynamics. The values for the parameters were carefully chosen to represent fairly clinical data in conformity with the condition for existence and uniqueness as well as stability criterion. The following initial data and control parameters were suggested;

$$\text{initial data}: \ S(0) = 5, \ T(0) = 4, \ T_i(0) = 1.5, \ v(0) = 0.25, \ P(0) = 1$$

$$\text{control parameters}: \begin{cases} \alpha_1 = 0.1, \alpha_2 = 0.6, \ \alpha_3 = 0.4, \ \alpha_4 = 0.25, \ \eta = 0.3, \mu = 0.5, \\ \delta = 0.2, \ \beta_1 = 0.3, \ \beta_2 = 0.1, \ \beta_3 = 0.2, \ \beta_4 = 0.15, \ \beta_5 = 0.3 \end{cases}$$

Using the above information, the graphical representation of the dynamics is as shown in Figs. 3 and 4 below

Immune responses and recovery during hiv treatment

The basic model dynamics given by equations (1) – (6) may not be an explicit representation of the hemodialysis treatment of HIV infections. However, the model fit accurately the viral kinetic data obtained during both natural infection and

when patients are on hemodialysis treatment. Fig. 3 shows that stem cell production depletes rapidly during the first phase of the infection. During this phase, the T Cell increases for a while to a maximum point at the early stage of the infection which later decline due to insufficiency of stem cell. Healthy T Cells decreases while infected cell and glycoprotein increases until the trio reaches equilibrium point after which further depletion of healthy T Cells resulted into decrease in infected cell and increase in glycoprotein gp120 concentration. From Fig. 4, it could be seen that availability of more stem cell mitigates the glycoprotein increase to around the initial concentration. Shortly when the healthy T Cells and infected cell moves out of phase, it resulted into decline in the number of infected cell. This is accompanied by decrease in the viral load as a result of that, the infected cell becomes latent, and stem cells and healthy T Cell start increasing. We observed that if this condition is sustained, there is possibility if total viral clearance and thus prevent migration to full blown AIDS.

Conclusion

Mathematical Analysis of Affinity Hemodialysis on T-Cell Depletion has led to a number of important insights about the dynamics of HIV infection. We have seen the possibility of more than two equilibriums which includes both disease free and infection equilibrium. The disease free equilibrium is stable. Thus, if the conditions suggested are superimposed on the dialysis treatment, then AIDS could be prevented. The first infected equilibrium is stable provided the rate of stem cell production is kept higher that the clearance rate due to death of the stem cell, while the second infected equilibrium is unstable and may lead to full blown AIDS. To ensure stability of all the infected points, it is necessary to ensure that glycoprotein is prevented from unnecessary increase and the stem cell is not allowed to deplete. Also recovery rate through hemodialysis should continuously ensure to increase. This work, therefore has open the way for collaborations between modelers and clinicians to further understand the nature and process of multiphasic viral decay observed in some treated patients, the initial expansion of virus, infected cells and glycoprotein as well as possibility of total viral clearance as depicted in Figs. 3 and 4. Effective and close interdisciplinary collaborations will encourage qualitative and quantitative modeling.

Declaration of Competing Interest

All authors have agreed and approved the manuscript and have contributed significantly towards the article. There is no conflict of interest among the authors.

References

[1] K.A. Lisowska, M. Pindel, K. Pietruczuk, I. Kuźmiuk-Glembin, H. Storoniak, J.M. Witkowski, The influence of a single hemodialysis procedure on human T lymphocytes, Sci Rep 9 (2019) 5041.
[2] J. Cohen, Shots in the Dark, The Wayward Search For an AIDS Vaccine, W.W. Norton and Co., New York, 2001, p. 384.
[3] S. Menzo, P. Bagnarelli, A. Monachetti, L. Fiorelli, M. Clementi, "Complexity and dynamics of HIV-1 quasispecies, Journal of Biological Regulatory and Homeostatic Agents 14 (2000) 4–6.
[4] S.I. Oke, S.O. Salawu, M.B. Matadi1, I.L. Animasaun, "Analysis of nonlinear radiative microwave heating of hyperthermia tumor cells therapy in a porous medium, Commun. Math. Biol. Neurosci. 19 (2019) 30.
[5] R.C. Gallo, J. Schupbach, Human retroviruses, in: S. Specter, R.L. Hodinka, S.A. Young (Eds.), Clinical Virology Manual, ASM Press, Washington, DC, 2000, pp. 513–549.
[6] A. Mocroft, S. Madge, A.M. Johnson, A. Lazzarin, N. Clumeck, F.D. Goebel, J.P. Viard, J. Gatell, A. Blaxhult, J.D. Lundgren, "A comparison of exposure groups in the EuroSIDA study: starting highly active antiretroviral therapy (HAART), response to HAART, and survival, Journal of Acquired Immunodeficieny Syndrome 22 (1999) 369–378.
[7] D.B. Clifford, C. Yiannoutsos, M. Glicksman, D.M. Simpson, E.J. Singer, P.J. Piliero, C.M. Marra, G.S. Francis, J.C. McArthur, K.L. Tyler, A.C. Tselis, N.E. Hyslop, "HAART improves prognosis in HIV-associated progressive multifocal Leukoencephalopathy, Neurology 52 (1999) 623–625.
[8] K. Mai, A. Boldt, H.M. Hau, M. Kirschfink, S. Schiekofer, F. Keller, J. Beige, A. Giannis, U. Sack, F.M. Rasche, "Immunological Alterations due to Hemodialysis Might Interfere with Early Complications in Renal Transplantation, Analytical Cellular Pathology 3 (2019) 67–74.
[9] S.E. Baum, J.T. Morris, R.V. Gibbons, R. Cooper, "Reduction in human immunodeficiency virus patient hospitalizations and non-traumatic mortality after adoption of highly active antiretroviral therapy, Mil Med 164 (1999) 609–612.
[10] B. Ramratnam, J.E. Mittler, L. Zhang, D. Boden, A. Hurley, F. Fang, C.A. Macken, A.S. Perelson, M. Markowitz, D.D. Ho, "The decay of the latent reservoir of replication-competent HIV-1 is inversely correlated with the extent of residual viral replication during prolonged anti-retroviral therapy, Nat. Med. 6 (2000) 82–85.
[11] S. Aquaro, R. Calio, E. Balestra, P. Bagnarelli, A. Cenci, A. Bertoli, B. Tavazzi, D. Di Pierro, M. Francesconi, D. Abdelahad, C.F. Perno, "Clinical implications; of HIV dynamics and drug resistance in macrophages, Journal of Biological Regulatory and Homeostatic Agents 12 (1998) 23–27.
[12] D. Wonder, A.L. Lloyd, V.A. Jansen, M.A. Nowak, "Dynamics of; macrophage and T cell infection by HIV, J. Theor. Biol. 196 (1999) 101–113.
[13] H.C. Tuckwell, E. Le Corfec, "A stochastic model for early HIV-1 population dynamics, J. Theor. Biol. 195 (1998) 451–463.
[14] J. Xiaoyan, C. Rongyi, C. Xuesen, Z. Jianzhou, J. Jun, D. Xiaoqiang, Y. Xiaofang, The difference of T cell phenotypes in end stage renal disease patients under different dialysis modality, BMC Nephrol 20 (2019) 301.
[15] M.A. Stafford, L. Corey, Y. Cao, E.S. Daar, D.D. Ho, A.S. Perelson, "Modeling plasma virus concentration during primary HIV infection, J. Theor. Biol. 203 (2000) 285–301.
[16] R.H. Tullis, D.O. Scamurra, Julian L. Ambrus, Affinity Hemodialysis for Antiviral Therapy with Specific Application to HIV, Journal of Theoretical Medicine (3) (2002) 157–166.
[17] D.A. Hildeman, T. Mitchell, J. Kappler, P. Marrack, T cell apoptosis and reactive oxygen species, J. Clin. Invest. 111 (2003) 575–581.
[18] B.A. Véronique, R. Hebmann, C. Devaux, M. Biard-Piechaczyk, Apoptosis of uninfected; cells induced by HIV envelope glycoproteins BioMed Central, Retrovirology 1 (2004) 12.
[19] R.O. Ayeni, A.O. Popoola, J.K. Ogunmoyela, Some new results on affinity; hemodialysis and T cell recovery, Journal of Bacteriology Research 2 (1) (2010) 001–004.
[20] A. Bandera, G. Ferrario, M. Saresella, I. Marventano, A. Soria, F. Zanini, F. Sabbatini, M. Airoldi, G. Marchetti, F. Franzetti, D. Trabattoni, M. Clerici, A Gori, CD4+; T Cell Depletion, Immune Activation and Increased Production of Regulatory T Cells in the Thymus of HIV Infected Individuals, PLoS ONE 5 (5) (2010) e10788.

[21] G. Doitsh, N.L. Galloway, X. Geng, Z. Yang, K.M. Monroe, O. Zepeda, P.W. Hunt, H. Hatano, S. Sowinski, I. Muñoz-Arias, W.C. Greene, Pyroptosis drives CD4 T-cell depletion in HIV-1 infection, Nature 23 (2014) 509–514.
[22] S. Keck, M. Schmaler, S. Gantera, L. Wyssa, S. Oberle, E.S. Husebyd, D. Zehnc, C.G. Kinga, Antigen affinity and antigen dose exert distinct influences on CD4 T-cell differentiation, PNAS 111 (41) (2014) 14852–14857.
[23] Hojo-Souza N.S., Pereira D.B., Mendes T.A., Passos L.S.A., Gazzinelli-Guimarães A.C., Gazzinelli-Guimarães P.H., Tada M.S., Zanini G.M., Bartholomeu D.C., Fujiwara R.T., Bueno L.L. (2015): CD4+ T cells apoptosis in Plasmodium vivax infection is mediated by activation of both intrinsic and extrinsic pathways Malaria Journal, 14, 5–13.
[24] R.J. Martinez, B.D. Evavold, Lower affinity T cells are critical components and active participants of the immune response, Front. Immunol. 6 (2015) 468.
[25] Y. Konagaya, R. Miyakawa, M. Sato, A. Matsugami, S. Watanabe, F. Hayashi, T. Kigawa, C. Nishimura, Effect of Glu12-His89 Interaction on Dynamic Structures in HIV-1 Matrix Protein Elucidated by NMR, PLoS ONE 11 (12) (2016) e0167176.